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$\boxed{1.}$ $f(\underset{\sim}{x};\theta) = \prod_{i=1}^{n} (2\pi\sigma_i^2)^{-1/2} \exp\left\{-\frac{1}{2\sigma_i^2}(x_i - a_i\theta)^2\right\}$

(a) $= (2\pi)^{-n/2} \times \prod_{i=1}^{n} (\sigma_i^2)^{-1/2} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{(x_i - a_i\theta)^2}{\sigma_i^2}\right\}$,

$$= \left.(2\pi)^{-n/2} \times \prod_{i=1}^{n} (\sigma_i^2)^{-1/2} \times \exp\left(-\frac{1}{2}\sum_{i=1}^{n}\frac{x_i^2}{a_i^2}\right)\right\} \text{parte de } h(\underset{\sim}{x}).$$

$$\underbrace{\times \exp\left(\theta\sum_{i=1}^{n}\frac{a_i x_i}{\sigma_i^2} - \frac{1}{2}\theta^2\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2}\right)}_{g(t,\theta)}$$

$g(t,\theta)$, com $t = \sum_{i=1}^{n}\frac{a_i x_i}{\sigma_i^2} \Rightarrow T = \sum_{i=1}^{n}\frac{a_i X_i}{\sigma_i^2}$ é suficiente.

$$l(\theta) = c_0 + \theta \times t - \frac{1}{2}\theta^2\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2}$$

$$\frac{\partial l(\theta)}{\partial\theta} = t - \theta\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2} \qquad \frac{\partial l}{\partial\theta} = 0 \Rightarrow \theta = \frac{t}{\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2}}$$

$$\frac{\partial^2 l(\theta)}{\partial\theta^2} = -\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2} < 0 \Rightarrow \hat{\theta} = \frac{\sum_{i=1}^{n}\frac{a_i X_i}{\sigma_i^2}}{\sum_{i=1}^{n}\frac{a_i^2}{\sigma_i^2}}$$

(b) 1º momento

$$E(X_i) = a_i\theta, \quad i = 1, \ldots, n$$

$$\frac{1}{n}\sum_{i=1}^{n} a_i\theta = \frac{1}{n}\sum_{i=1}^{n} X_i \Rightarrow \hat{\theta} = \frac{\bar{X}}{\bar{a}}$$

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