

Resolução da Recorrência

$$T(n) = 2T(n/2) + 1$$

$$T(1) = 1$$

$$T(n) = 2T(n/2) + 1 \quad i = 1$$

$$2. \cancel{T(n/2)} = 2. \cancel{2T(n/4)} + 1. 2 \quad i = 2$$

$$4. \cancel{T(n/4)} = 4. \cancel{2T(n/8)} + 1. 4 \quad i = 3$$

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$$2^{k-1}. \cancel{T(2)} = 2^{k-1}. \cancel{2T(1)} + 1. 2^{k-1} \quad i = \log n$$

$$2^k. \cancel{T(1)} = 2^k. 1$$

Genericamente, tem-se:

$$T\left(\frac{n}{2^{k-1}}\right) = 2T\left(\frac{n}{2^k}\right) + 1$$

$$\text{Para } T(2) = \frac{n}{2^{k-1}} = 2 \Rightarrow a = 2^k \Rightarrow k = \log n$$

Somando:

$$T(n) = 2^k + \sum_{j=0}^k 2^j - 1$$

$$k = \log n \Rightarrow T(n) = n + \sum_{i=0}^{\log n - 1} 2^i - 1$$

$$n + 2^{\log n} - 2 - 1$$

$$2n - 3$$