Complex Neural Networks - Experiments with Collective Dynamics of Neuron Models

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Abstract

This report will discuss the experiments conducted in collective dynamics. As a topic for the tests, was chosen neuron models (FitzHugh-Nagumo and Hindmarsh-Rose models). But actually, any kind of dynamics could be used instead (e.g., oscillators). First, it will be shown that discretization problems appear when trying to simulate these dynamics in computers. Afterward, experiments will be conducted with single neurons to illustrate their dynamics and theoretical explanations will be drawn based on the phase portraits of both FitzHugh-Nagumo and Hindmarsh-Rose models. Thus, we conclude with an empirical analysis over a system composed of three neurons, which exhibit unexpected behavior.

1 Single Neuron Tests

The following compose tests executed over single model of neurons. The objective is understanding the dynamics as well as the analysis of the resulting behavior of modifications in the parameters of the model. In these tests we will focus on the FitzHugh-Nagumo and Hindmarsh-Rose models.

Next, tests over these single neurons will be shown as well as their definitions and interesting biological properties. But before going deep into these subjects we would like to cover the problems from the computing simulations.

1.1 Problems of the Simulations

To test these models, their equations were programmed in C++. But although simple, this process can be troublesome. The hurdle is that the neuron models are defined by differential equations over the continuous. And therefore, to simulate them it is necessary to realize a discretization stage.

Which consists of basically multiplying all the equations by a discretization period (a number between 0 and 1). The failure of doing this process leads basically to unstable equations which oscillate in the case of the Hindmarsh-Rose model between $+\infty$ and $-\infty$.

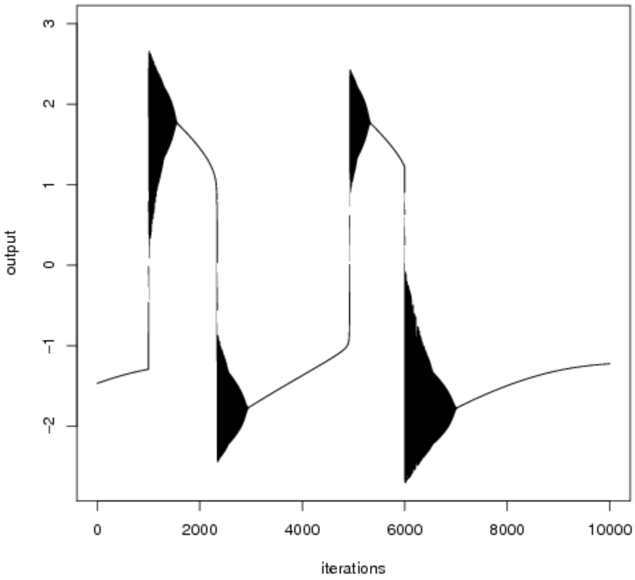

Figure 1: FitzHugh-Nagumo Model with discretization problems. The discretization period is only 0.9. Other variables are set respectively to $a = 0.7, b = 0.8, \tau = 1500, I = 0.35$.

But multiplying a simple discretization period is also not as easy as it sounds. Observe the Figure 1. Notice that in all the spikes the peaks and valleys oscillate. This shows that when the discretization period is not big enough the model do not respond well. A more evident example of this problem is shown on Figure 2, where the Hindmarsh-Rose is plotted. This time the discretization period is set to 0.1 and the model is still suffering from discretization problems, as can be seen between the 6000 and 8000 iterations. Note that the numbers appearing in the x-coordinate of all the Figures are the number of iterations, which correspond to the time multiplied by the discretization period.

To solve the problem of the Figure 1 the discretization period was decreased to 0.1 (check Figure 3) and for the problem shown on Figure 2 the discretization period was decreased to 0.01. This result is plot on the Figure 4. Notice that the wave pattern changes this time, even with the param-

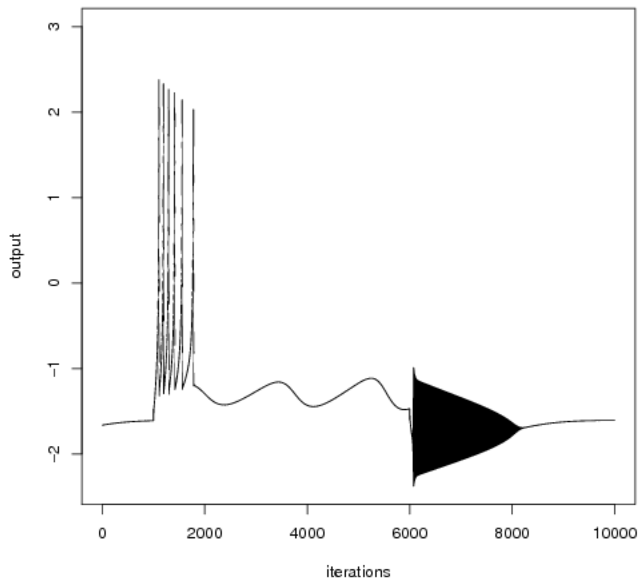

Figure 2: Hindmarsh-Rose Model with discretization problems. The discretization period is already 0.1 and the model is still not producing correct results. The other variables are set respectively to $a = 5.0, b = 3.0, r = 0.005, s = 4.0, x_r = -1.6, I = 1.35$.

eters being kept the same. This illustrates the dangerous interference of a bad discretization in the simulation of continuous dynamics and different necessity of precision in different dynamics (FitzHugh-Nagumo is less sensible to the discretization than the Hindmarsh-Rose model).

1.2 FitzHugh-Nagumo Neuron Model

The FitzHugh-Nagumo (FN) model is a simple neuron model derived from a simplification of the Hodgin-Huxley (HH) model, which exhibits some biologically plausible characteristics [1], [5]. Therefore, sometimes it is used for explaining Hodgin-Huxley characteristics. It is exactly defined by the following equation:

$$\begin{aligned} \frac{dv}{dt} &= v - \frac{v^3}{3} - w + I, \\ \frac{dw}{dt} &= \frac{1}{\tau}(v + a - bw), \end{aligned} \tag{1}$$

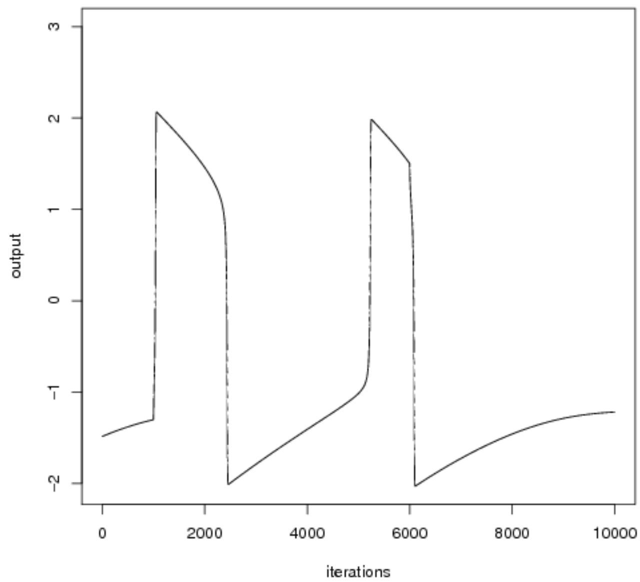

Figure 3: FitzHugh-Nagumo Model with correct discretization. The discretization period is now 0.1, the other variables are the same as Figure 1, setting aside the τ. Which was decreased to $\frac{1500}{9}$ in the same proportion of the discretization, to keep the same visual pattern.

where τ determines the period of the output, v is the membrane potential, w is its recovery variable, I is the current stimulus input and a, b are parameters that modify the dynamics.

For the following experiments it will be used a input wave of the form despicted in Figure 5 with the value of I varying according to the experiment. This model exhibit a sub and suprathreshold responses (respectively Figure 6 and 3 are examples of sub and suprathreshold). Another interesting behavior is the refractory period which occurs right after the spike and the periodic spiking. Both can be seen in Figure 3. All of these dynamics can be also understood in the light of a phase portrait (Figure 7). To create this phase portrait it is draw the nullclienes ($\frac{dv}{dt} = 0$ and $\frac{dw}{dt} = 0$). And the resting point is the point where the state remains the same in both equations, or in other words, where they cross each other in the plane. The meaning of this is that in the resting point, the variation of the system is 0 (which implies that their derivatives are 0) and the values of the state (set of variables which determine the orbit of the system) does not change, therefore we substitute one equation in the other to find the resting point(s). In the case of the FN

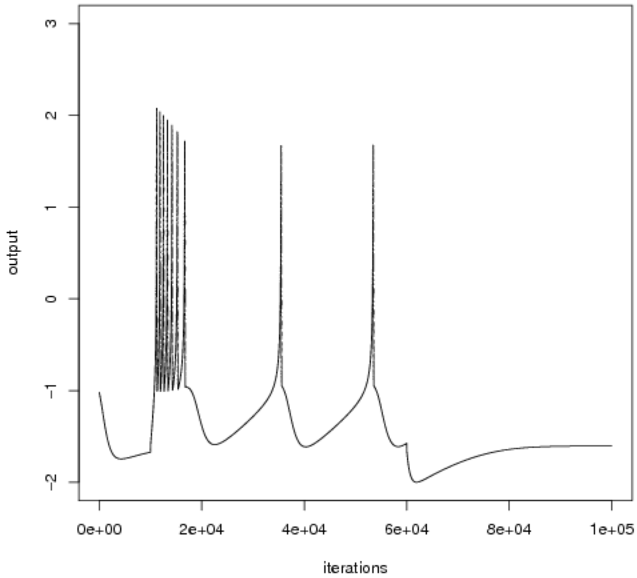

Figure 4: Hindmarsh-Rose Model with correct discretization. The discretization period is now 0.01, the other variables are the same as Figure 2. Setting aside the τ, which the number of iterations which increased in the same proportion of the discretization, to keep the same visual pattern.

model, there is just one resting point.

The FN model has various biological related deficiencies [3]. In relation to these deficiencies, we can cite the following:

- Lack of bursting (rapid firing);
- Small number of parameters which make it difficult to adapt to specific neurons.

And it explain some phenomenons observed in practice, such as:

- Excitation Block: block in the periodic spiking caused by the increase of I;
- Anodal Break Excitation: a spike that occurs when the system is hyperpolarized (I is negative) and then is released from hyperpolarization (for example, $I = 0$).

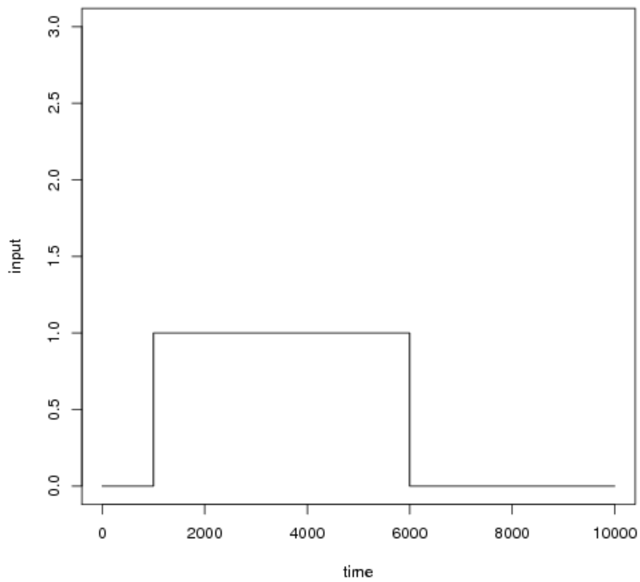

Figure 5: Input wave used in the experiments.

1.3 Hindmarsh-Rose Neuron Model

The Hindmarsh-Rose (HR) model solves some of the deficiencies presented by the FN model, but adds new complexity to it making the analysis more difficult [2]. It is defined by the following equation:

$$\begin{aligned}\frac{dx}{dt} &= y - x^3 + bx^2 - z + I,\\ \frac{dy}{dt} &= 1 - ax^2 - y,\\ \frac{dz}{dt} &= r[s(x - x_r) - z],\end{aligned} \tag{2}$$

where x is the membrane potential, the y is the spiking variable which corresponds to the sodium and potassion channels, z is the bursting variable which accounts other slow channels, x_r is the resting value of x, I is the current applied and s, r, a, b are other constants.

One of the main characteristics of this model is the presence of bursting, which is characterized by the rapid fire of a neuron. Figure 8 show the neuron with this behavior. Another interesting biological phenomenon is shown on Figure 9, which is called adaptation. The adaptation is defined as a modulation of the frequency with respect to the magnitude of the input.

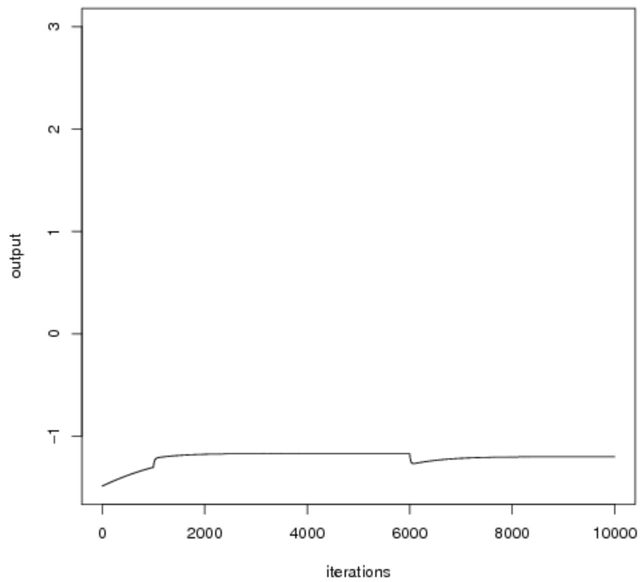

Figure 6: With $I = 0.05$ the input was not enough to produce a spike. This is called subthreshold response.

The input signal in this case is a ramp from 0.25 to 9.0, which increase quadratically between the iteration 10000 and 60000.

In the same way as the FN model, the HR model can also be described by its phase portrait (Figure 10). Where the thin lines are the nulclines and the thick lines are the limit-cycle (closed trajectory which other trajectories spirals into). Here we will not enter into the details but rather give a brief overview. There are 3 equilibrium points: the stable point A, the saddle point B and the unstable point C. Basically, the system dynamics state at rest on the point A. Then, when a sufficient current is applied the x-nulcline is lowered and both points A and B disappear. Resulting in a limit-cycle which is responsible for the spikes.

2 Test with Coupled Neurons

Consider two neurons a and b receiving inputs $I1$ and $I2$ and connected to a third neuron c by their x variable (x-coupled). This topology is shown in Figure 11. Considering the case where both inputs are the same, when both neurons (a, b) receive a wave with intensity 1.5. Both neurons produce spike,

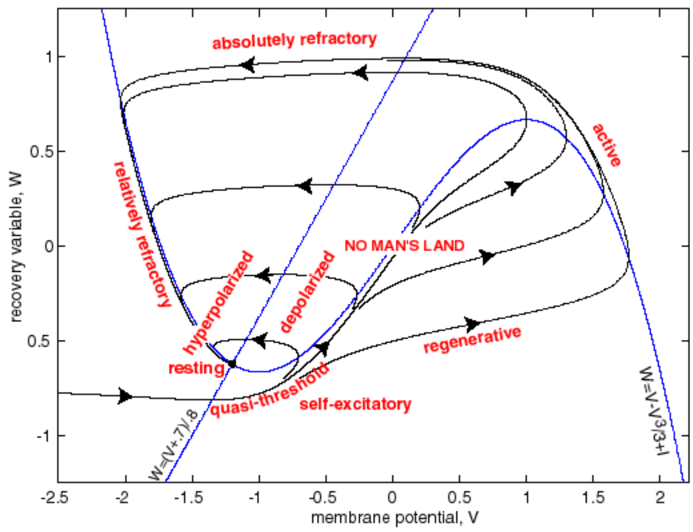

Figure 7: Phase portrait of the FN model. From the [4].

as seen before. But the spike received simultaneously by the next neuron would not activate a spike, as shown in Figure 12.

In the case of both input being 1.7, we can note the increasing in amplitude of the output (Figure 13). The peak increases steeply with the increase in the input from 1.7 to 2.0, and finally at 2.0 it became a spike (Figure **??**). Therefore, it can be said that the neuron c has a spike which is only activated with an intensity of 2.0 from the input. Which is much higher than its natural firing threshold (around 0.5).

Now, let the inputs keep the same but with a delay of 30 miliseconds. Then the Figure 15 is observed. Which means the delay made little effect on the resulting spike. However when the delay is of 40 miliseconds Figure 16 is seen. Where no spike is present. We conclude that with a certain amount of delay, in this case 40 miliseconds, the neurons will desynchronize their outputs enough to eliminate the spike in the last neuron.

3 Conclusion

The first step for the simulation of collective dynamics is the simulation of the dynamics itself. Although seemly simple, the programming of dynamics in the computer hide some complicated issues. The discretization is a necessity and the discretization period can yet vary from problem to problem.

Two types of dynamics were reported, which are the FitzHugh-Nagumo and the Hindmarsh-Rose neuron models. Their differences and similarites

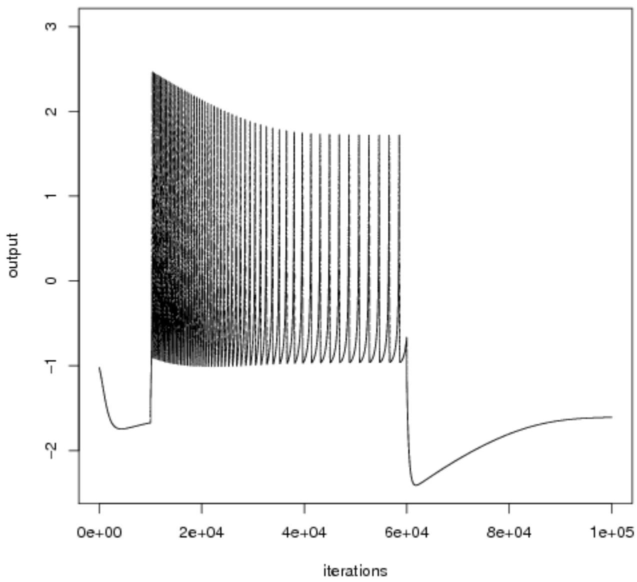

Figure 8: HR model exhibits bursting. Discretization step is 0.01, $a = 5.0, b = 3.0, r = 0.005, s = 4.0, x_r = -1.6, I = 4.0$. The input wave has the form of the Figure 5

were discussed with empirical tests and theoretical approaches.

This report also showed that complex dynamics can be more clearly understood in the light of phase portraits. But in this scenario, the knowledge of dynamical systems concepts become unavoidable and therefore was not reported in depth here. Notice also, that the same phase portrait approach could be extended to coupled systems.

Last, it was shown that the behavior of three coupled neurons exhibit unexpected behavior. Which can not be easily derived from their single dynamics. This simple example demonstrate the difficulty of analysing collective dynamics empirically. And suggested that maybe an ergodic approch should be used instead.

References

[1] R. Fitzhugh. Impulses and physiological states in theoretical models of nerve membrane. *Biophysical Journal*, 1(6):445–466, 1961.

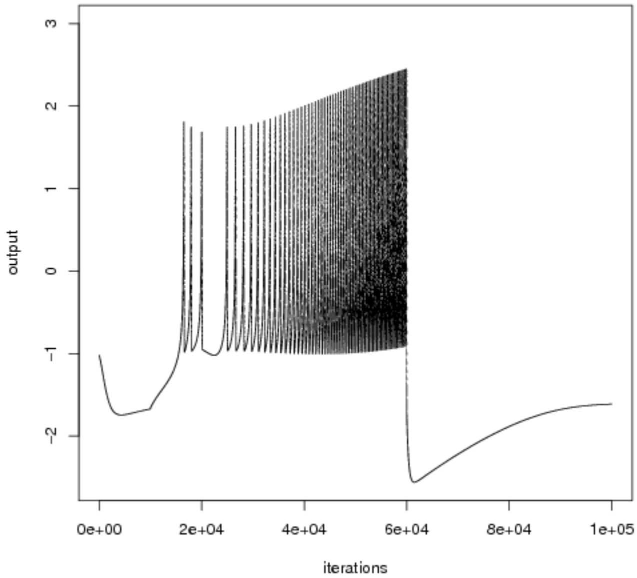

Figure 9: HR model adapting to the signal. Aside from the value of I, which varies with time, the remaining variables are set to the same value as the Figure 8.

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[3] E.M. Izhikevich. *Dynamical systems in neuroscience: The geometry of excitability and bursting.* The MIT press, 2007.

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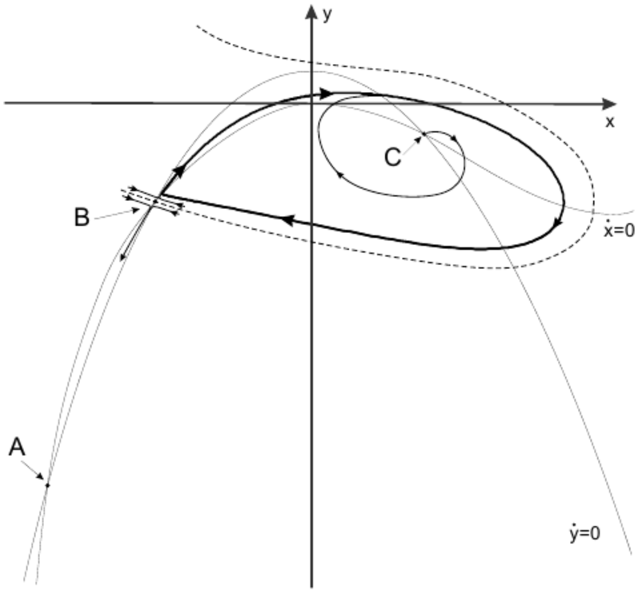

Figure 10: Phase portrait of the HR model. From the [6].

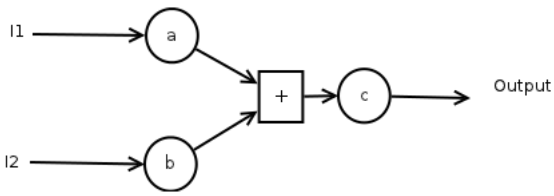

Figure 11: Topology of neurons used. The circles are HR neurons and the square is a simple operation.

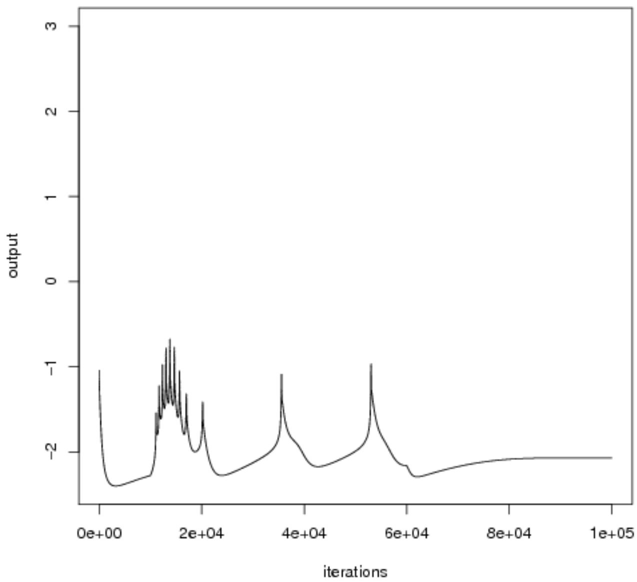

Figure 12: Output of neurons for the same input with intensity 1.5.

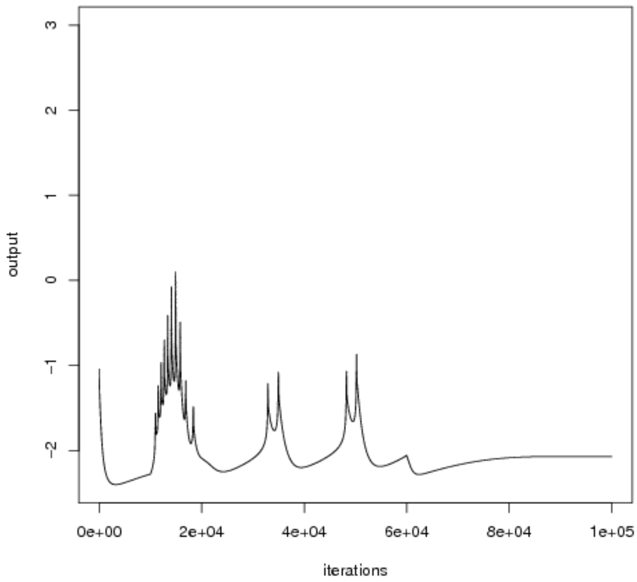

Figure 13: Output of neurons for the same input with intensity 1.7.

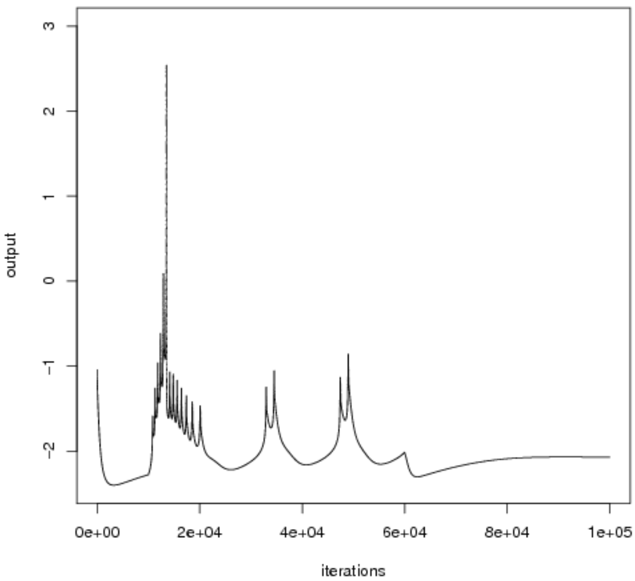

Figure 14: Output of neurons for the same input with intensity 2.0.

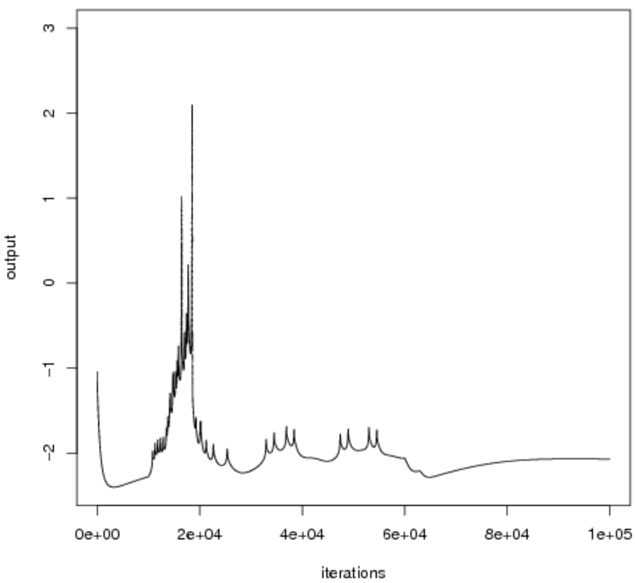

Figure 15: Output of neurons for the same input with intensity 2.0 and a 30 miliseconds (3000 iterations) delay between each other.

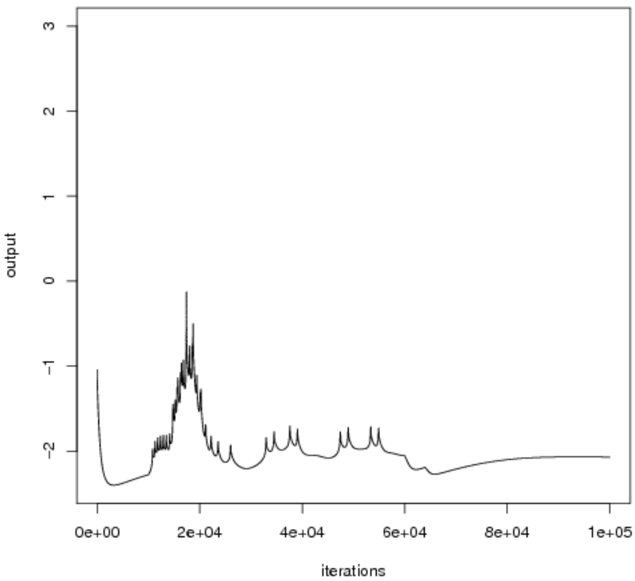

Figure 16: Output of neurons for the same input with intensity 2.0 and a 40 miliseconds (4000 iterations) delay between each other.