SMA 0333 Cálculo III - Lista 4

Eng. Aeronáutica

1. Utilizando o critério de Cauchy mostre que se a série $\sum_{n=0}^{\infty} a_n$ converge então $\lim_{n\to\infty} a_n = 0$

2. Uma bola é atirada de uma altura de 10m. Em cada instante, golpeia o chão e ricocheteia verticalmente a uma altura que é $\frac{3}{4}$ da altura precedente. Ache a distância total que a bola viajará se for admitindo que ricocheteia muitas vezes.

3. Suponhamos que $a_n > 0$ para todo inteiro $n \geq 0$ e $\sum_{n=0}^{\infty} a_n$ diverge então $\sum_{n=0}^{\infty} \frac{a_n}{1+a_n}$ também diverge.

4. Determine se as séries altenadas convergem. E determine se as mesmas convergem absolutamente.

$a)\ \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ $\quad b)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{3}{2}}}$ $\quad c)\ \sum_{n=0}^{\infty} (-1)^n \frac{2n^2+1}{n^3+3}$ $\quad d)\ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)}$

$e)\ \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln^2(n)}$ $\quad f)\ \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$ $\quad g)\ \sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln(n)}$ $\quad h)\ \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{\sqrt{n}}$

$i)\ \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$ $\quad j)\ \sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{2^n}}{n!}$ $\quad k)\ \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+1}$ $\quad l)\ \sum_{n=0}^{\infty} \frac{(-n)^n}{n!}$

$m)\ \sum_{n=1}^{\infty} (-1)^n \text{sen}\left(\frac{1}{n^p}\right),\ p \geq 1,\ p \in \mathbb{R}.$

5. Estude a convergência das séries:

$a)\ \displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\sin\frac{1}{n}$ $\quad b)\ \displaystyle\sum_{n=2}^{\infty}(-1)^{n}\frac{n^3}{n^4+3}$ $\quad c)\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$

$d)\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{3n+1}$ $\quad e)\ \displaystyle\sum_{n=3}^{\infty}\frac{\ln n}{n^2}$ $\quad f)\ \displaystyle\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$

$g)\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{3^{n-1}+2}$ $\quad h)\ \displaystyle\sum_{n=3}^{\infty}\frac{1}{\sqrt[n]{2}}$ $\quad i)\ \displaystyle\sum_{n=1}^{\infty}\frac{\arctan n}{1+n^2}$

$j)\ \displaystyle\sum_{n=0}^{\infty}\frac{n+1}{(n+2)2^n}$ $\quad k)\ \displaystyle\sum_{n=1}^{\infty}\frac{n+1}{\ln(n+2)}$ $\quad l)\ \displaystyle\sum_{n=1}^{\infty}\frac{n}{1+n^2}$

$m)\ \displaystyle\sum_{n=0}^{\infty}\frac{n^5+4n^3+1}{2n^8+n^4+2}$ $\quad n)\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+1)}{3n}$ $\quad 0)\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{2n+1}$

$p)\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\ln^2 n}{n}$ $\quad q)\ \displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\sqrt{n}\arctan\frac{1}{n+1}$ $\quad r)\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}-\frac{\pi}{4}\right)$

$s)\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2^{2(n-1)}}$ $\quad t)\ \displaystyle\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{3n}\frac{1}{(-3)^n}$ $\quad u)\ \displaystyle\sum_{n=0}^{\infty}\frac{n!}{(-2)^n}$

6. Estude a convergência das séries

$a)\ \displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n^2+3}$ $\quad b)\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{sen}\frac{1}{n}$ $\quad c)\ \displaystyle\sum_{n=0}^{\infty}ne^{-1}$ $\quad d)\ \displaystyle\sum_{n=2}^{\infty}\frac{1}{\ln(n)}$

$e)\ \displaystyle\sum_{n=0}^{\infty}\frac{2^n}{n!}$ $\quad f)\ \displaystyle\sum_{n=0}^{\infty}\frac{n!+n^2}{(n+1)!}$ $\quad g)\ \displaystyle\sum_{n=0}^{\infty}\frac{n^3}{3^n}$ $\quad h)\ \displaystyle\sum_{n=0}^{\infty}\frac{3^n}{1+4^n}$

$i)\ \displaystyle\sum_{n=0}^{\infty}\frac{n}{n^2+2n+1}$ $\quad j)\ \displaystyle\sum_{n=1}^{\infty}\frac{n^5-3n^4+2}{2n^8+n-1}$

7. A série $\displaystyle\sum_{n=0}^{\infty}na^n$, com $|a|<1$ é convergente? É absolutamente convergente? O que podemos dizer da série $\displaystyle\sum_{n=0}^{\infty}n^ra^n$, para r inteiro positivo? E se $|a|\geq 1$?

8. A série $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$, para um número real x fixado arbitrariamente, converge absolutamente?

9. Enuncie e demonstre o Critério da comparação por limites.

10. Enuncie e demonstre o Critério da Raiz por limites.