2

2. (a) $\theta_2 = 1$: $\sum X_i + \sum X_i^3$.

(b) $\theta_1 = 1$: $\sum X_i^2 + \sum X_i^3$.

(c) $(\sum X_i, \sum X_i^2, \sum X_i^3)^T$

teorema da fatoração

3. $f(\underset{\sim}{x};\theta) = \frac{1}{\theta^n} \cdot \left(\prod_{i=1}^{n} x_i\right)^{\frac{1-\theta}{\theta}} \times \prod_{i=1}^{n} I_{(0,1)}(x_i)$

(a)

$$= \underbrace{\frac{1}{\theta^n} \cdot \left(\prod_{i=1}^{n} x_i\right)^{1/\theta}}_{g(t;\theta)} \underbrace{\times \left(\prod_{i=1}^{n} x_i\right)^{-1} \times \prod_{i=1}^{n} I_{(0,1)}(x_i)}_{h(\underset{\sim}{x})}$$

$\Rightarrow T = \prod_{i=1}^{n} X_i$ é suficiente

(b) $f(x) = \frac{1}{\theta} x^{1/\theta} \cdot x^{-1} \cdot I_{(0,1)}(x)$

$= \exp\left\{\frac{1}{\theta}\log(x) - \log(\theta) - \log(x)\right\} I_{(0,1)}(x)$

$c(\theta) = \frac{1}{\theta}$, $t(x) = \log(x)$, $d(\theta) = -\log(\theta)$, $S(x) = -\log(x)$,

$A = (0,1)$.

(c) $\eta = \frac{1}{\theta}$ e $d_0(\eta) = -\log\left(\frac{1}{\eta}\right) = \log(\eta)$

EMV: $E_\eta[\log(X)] = -d_0'(\eta) = -\frac{1}{\eta}$

$\Rightarrow \hat{\eta} = -\frac{1}{\overline{\log(X)}} \Rightarrow \hat{\theta} = -\overline{\log(X)}$, $\overline{\log(X)} = \frac{1}{n}\sum_{i=1}^{n}\log(X_i)$.

de outra forma

$\ell(\theta) = c_0 - n\log(\theta) + \frac{1}{\theta}\sum_{i=1}^{n}\log(x_i)$

$\frac{\partial \ell(\theta)}{\partial\theta} = -\frac{n}{\theta} - \frac{1}{\theta^2}\sum_{i=1}^{n}\log(x_i)$; $\frac{\partial \ell(\theta)}{\partial\theta} = 0 \Rightarrow \hat{\theta} = -\overline{\log(x)}$

Provar que $\left.\frac{\partial^2 \ell(\theta)}{\partial\theta^2}\right|_{\theta=\hat{\theta}} < 0$.