

SCE 0110 -  
Elementos de Lógica Digital I

**Implementação otimizada  
de funções lógicas  
(continuação)**

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# *Sumário*

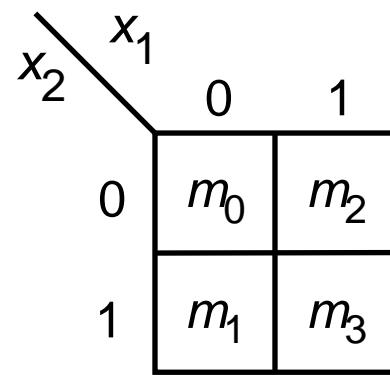
- Mapa de Karnaugh (Veitch-Karnaugh)
  - SOP (sum-of-products) e POS (product-of-sums)

## *- revisão da aula anterior -*

- O Mapa de Karnaugh é uma alternativa à tabela verdade para representação de funções

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



(b) Karnaugh map

Figure 4.2. Location of two-variable minterms.

## *- revisão da aula anterior -*

- A representação por mapa de Karnaugh facilita o reconhecimento de minitermos que podem ser combinados usando a propriedade 14a da Álgebra Booleana
- O resultado é a função mínima

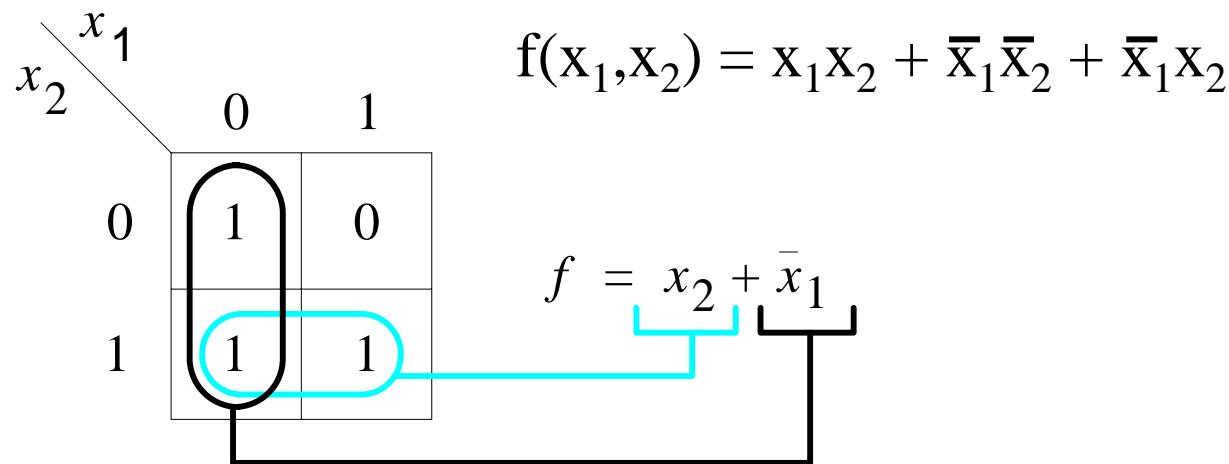
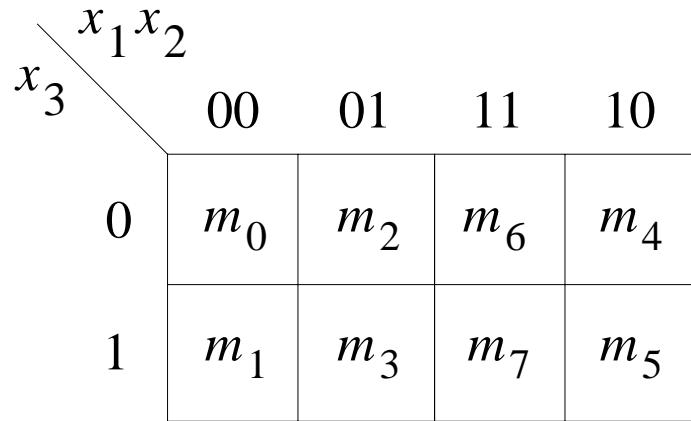


Figure 4.3. The function of Figure 2.15.

# *3 variáveis*

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

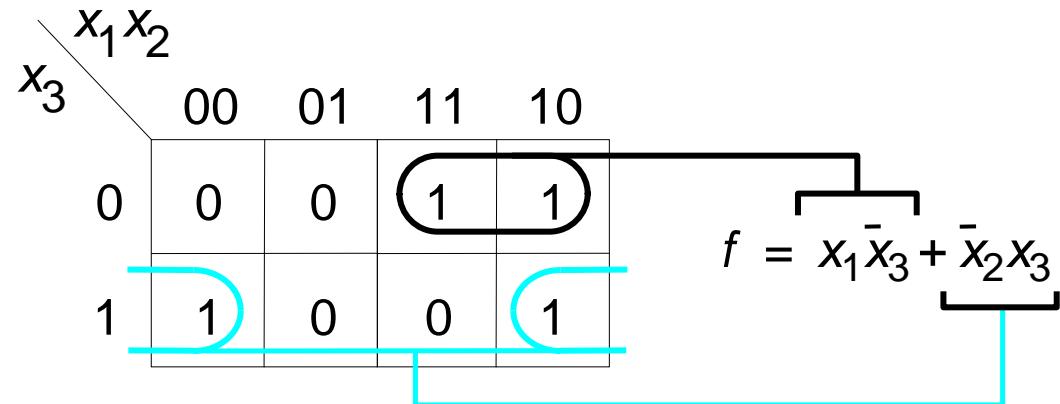
(a) Truth table



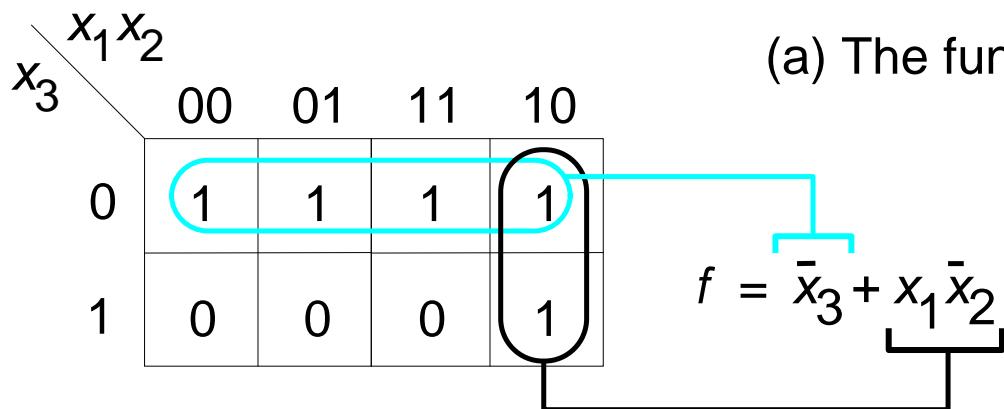
(b) Karnaugh map

Figure 4.4. Location of three-variable minterms.

- Basta cobrir todas as células, mas por que não unir a última coluna também?
- Variação de um bit por vez



(a) The function of Figure 2.18



(b) The function of Figure 4.1

Figure 4.5. Examples of three-variable Karnaugh maps.

## *4 variáveis*

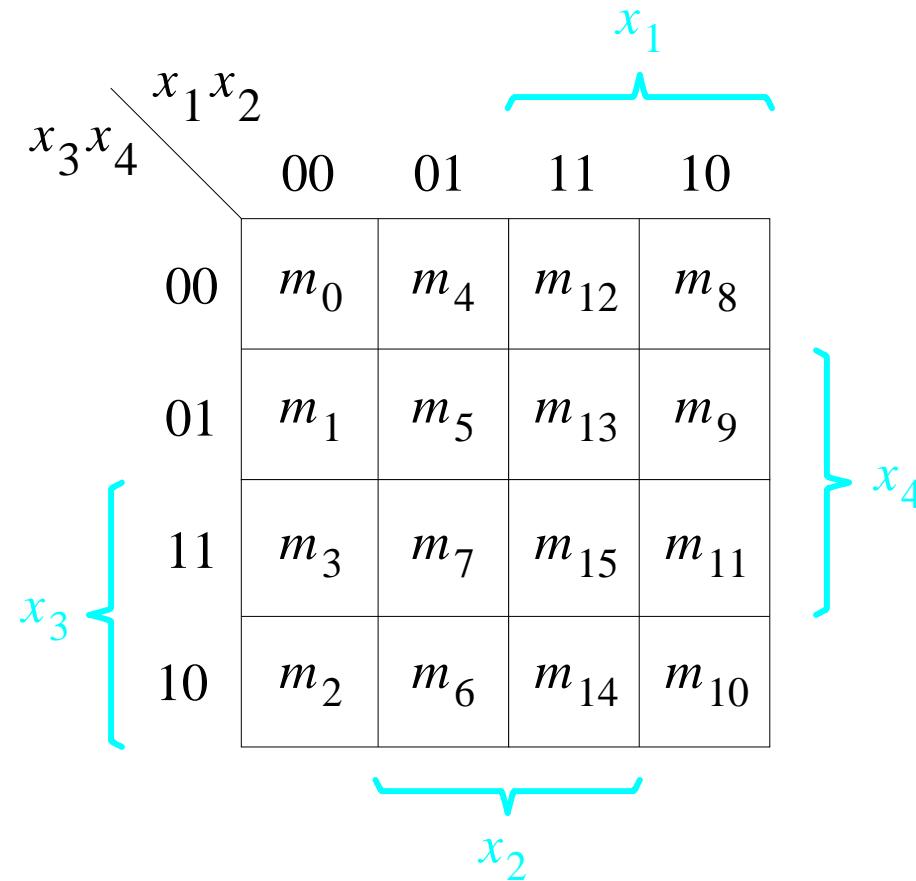


Figure 4.6. A four-variable Karnaugh map.

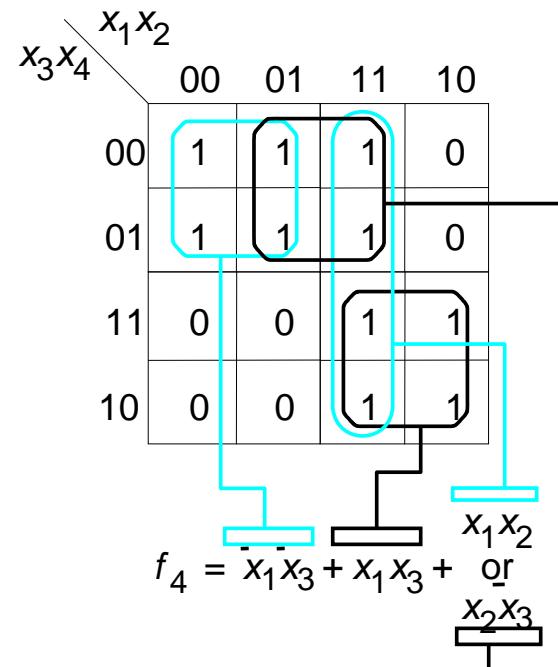
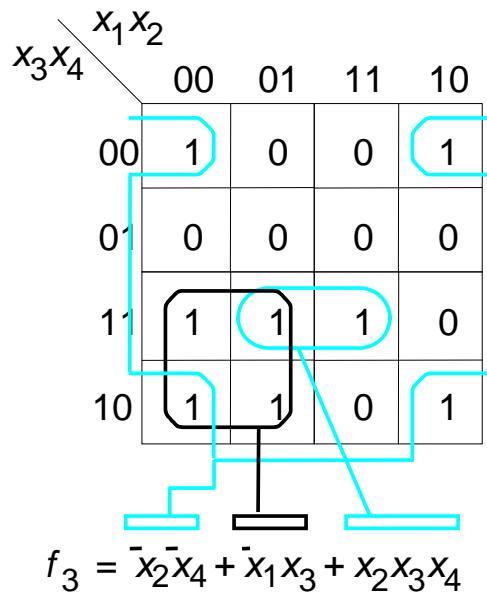
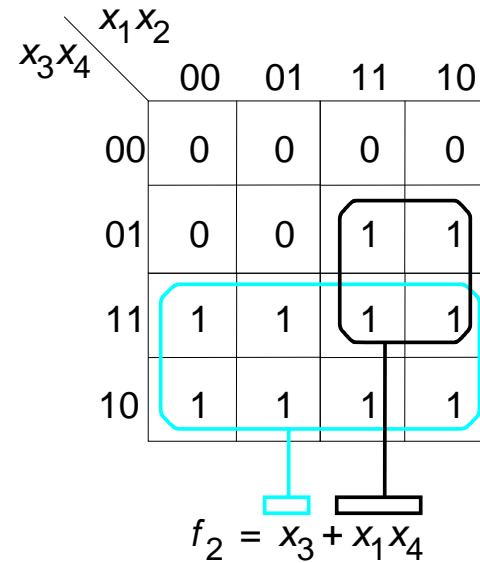
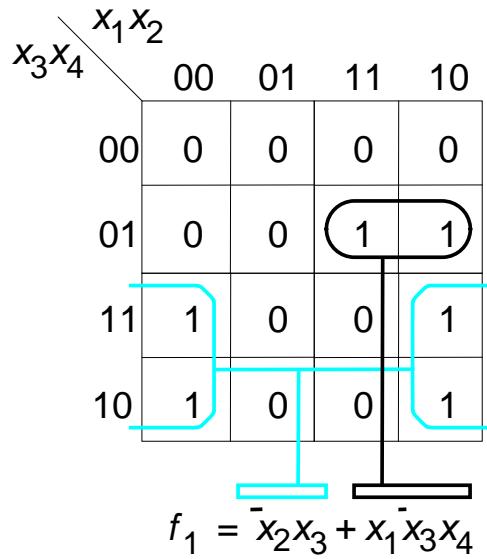


Figure 4.7. Examples of four-variable Karnaugh maps.

# *5 variáveis*

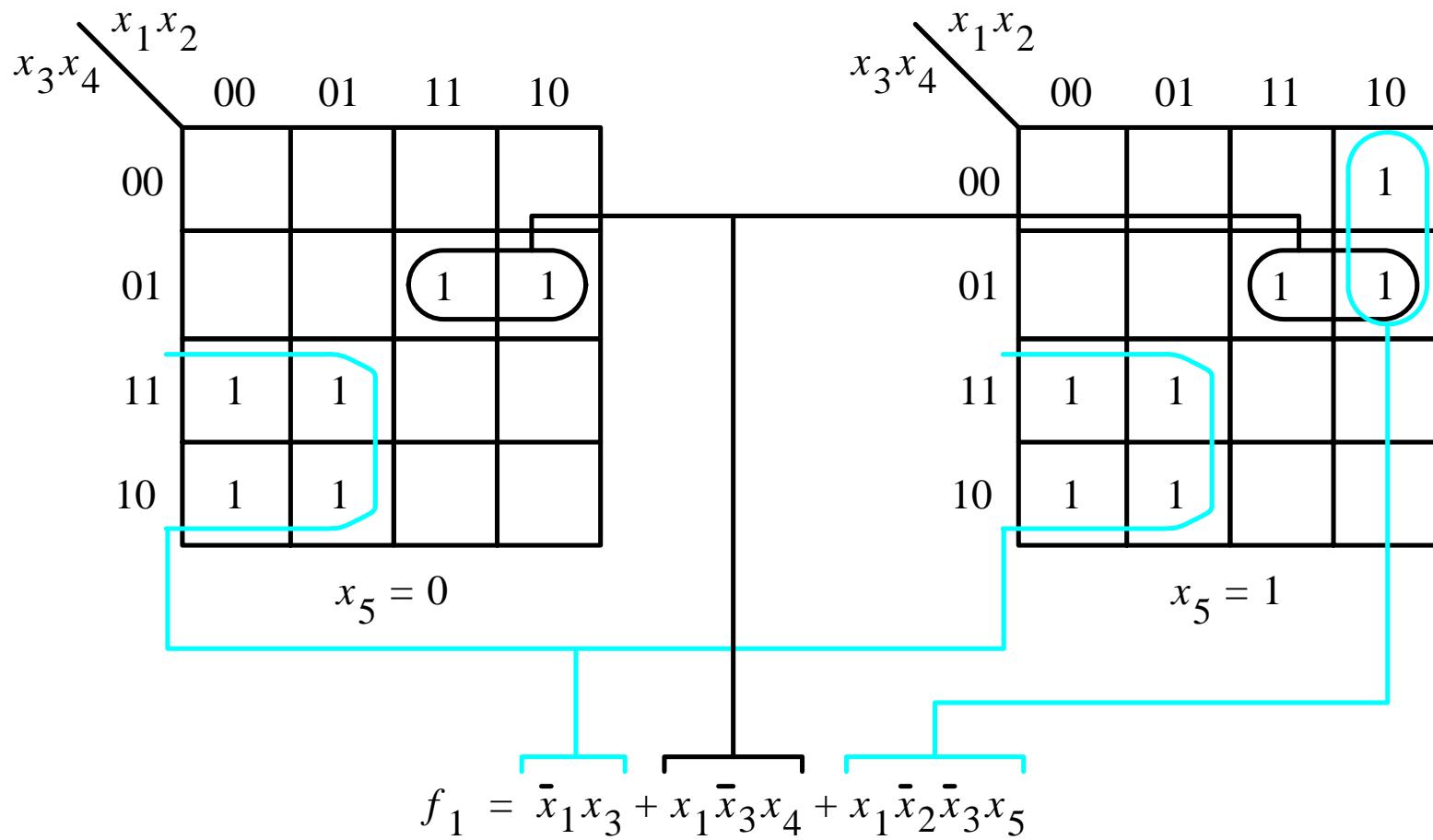


Figure 4.8. A five-variable Karnaugh map.

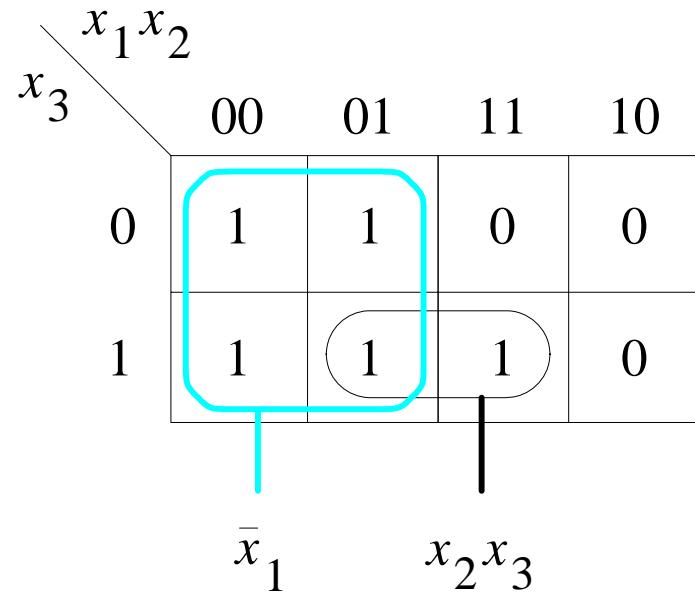


Figure 4.9. Three-variable function  $f(x_1, x_2, x_3) = \Sigma m(0, 1, 2, 3, 7)$ .

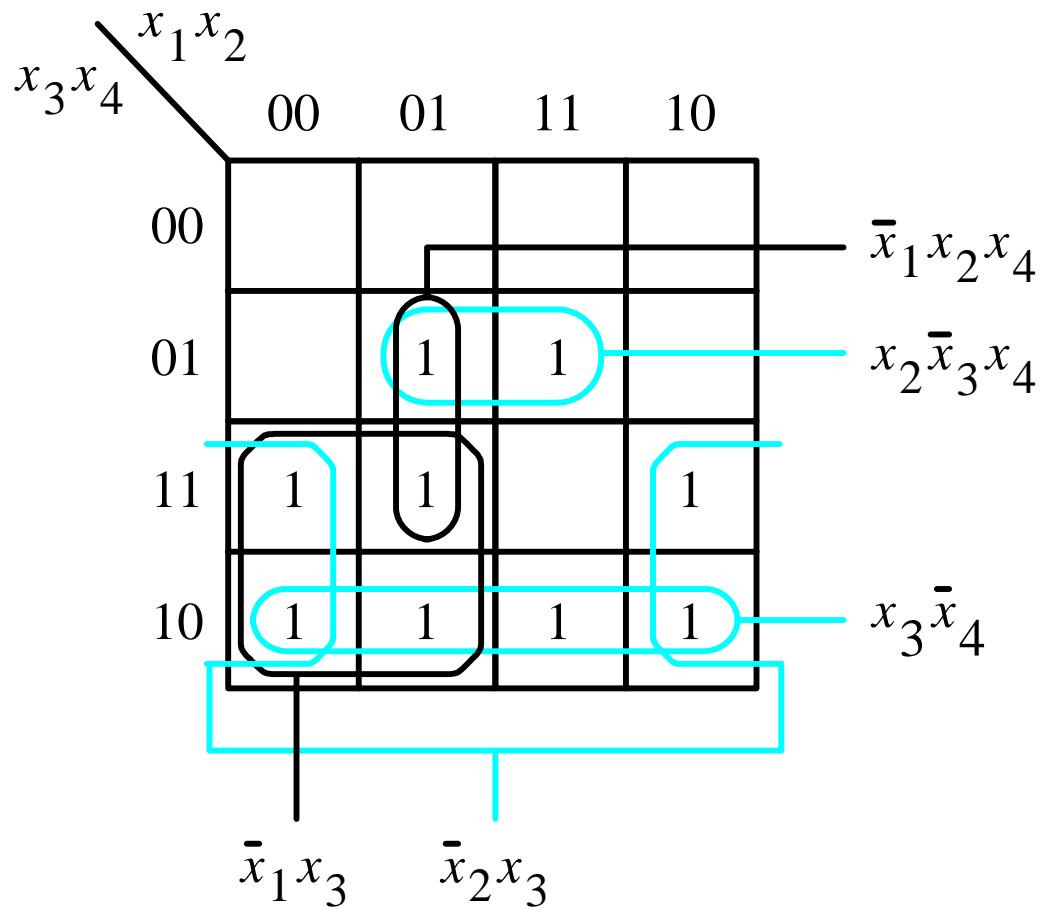


Figure 4.10. Four-variable function  $f(x_1, \dots, x_4) = \sum m(2, 3, 5, 6, 7, 10, 11, 13, 14)$ .

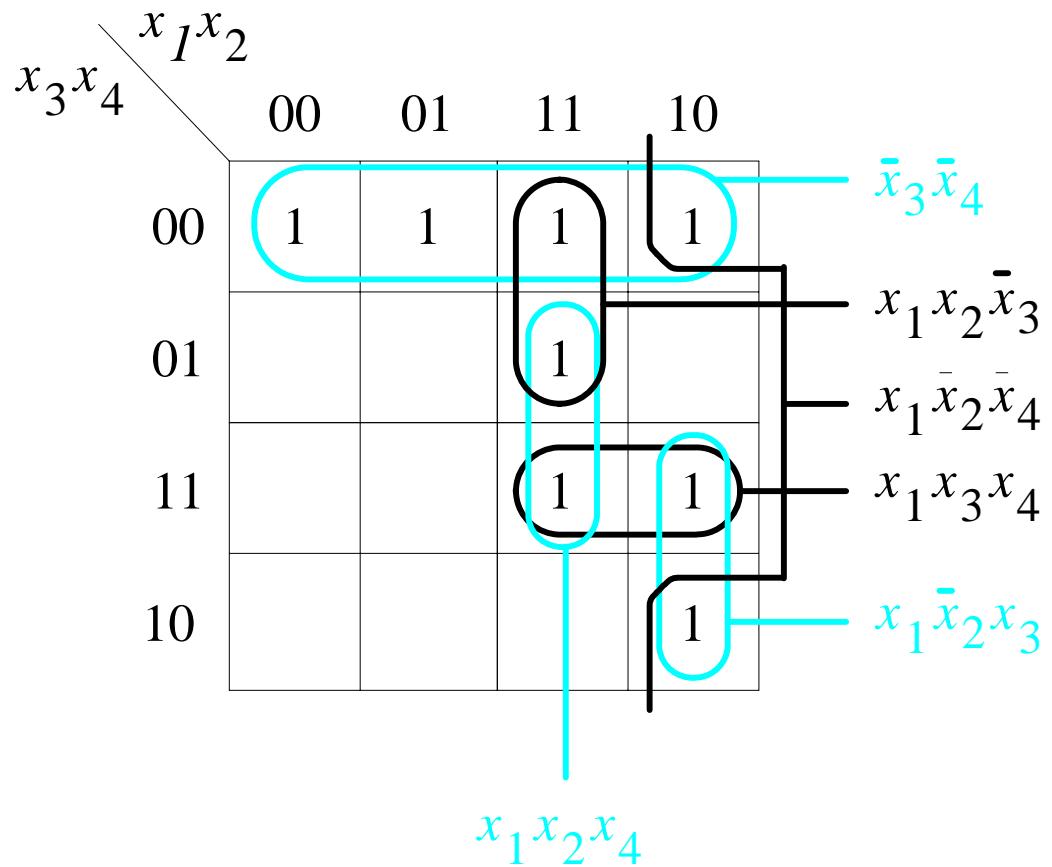


Figure 4.11. The function  $f(x_1, \dots, x_4) = \sum m(0, 4, 8, 10, 11, 12, 13, 15)$ .

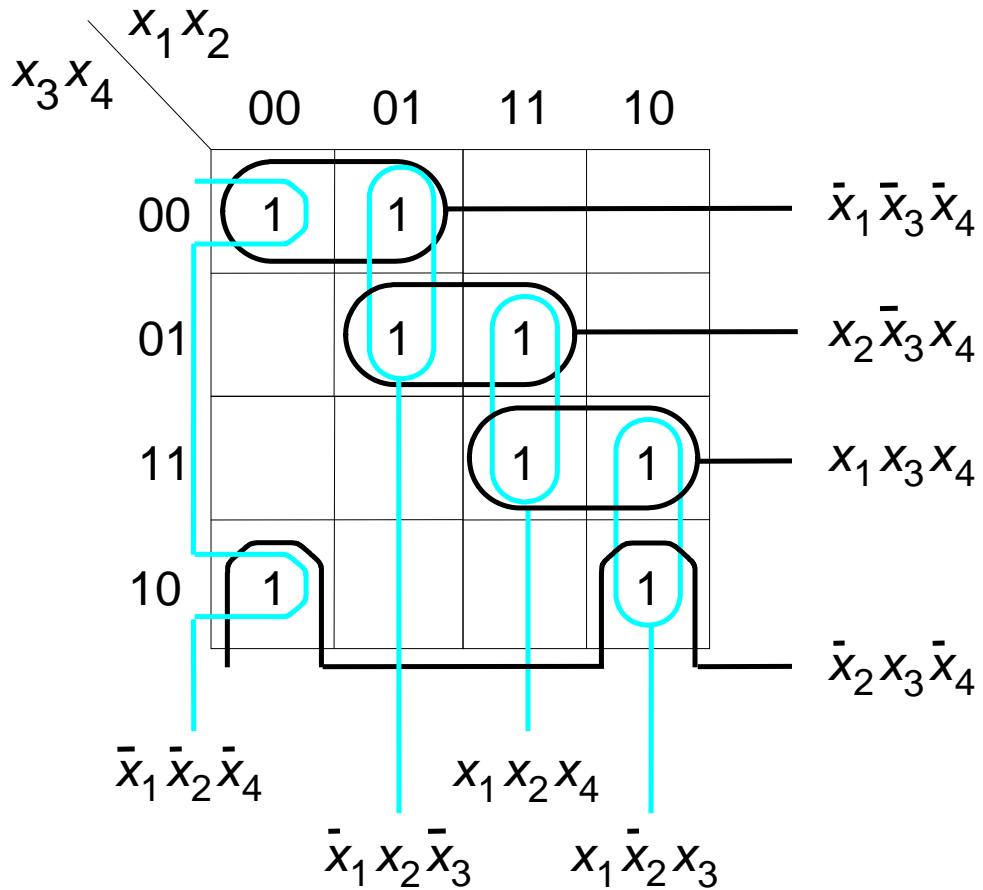


Figure 4.12. The function  $f(x_1, \dots, x_4) = \sum m(0, 2, 4, 5, 10, 11, 13, 15)$ .

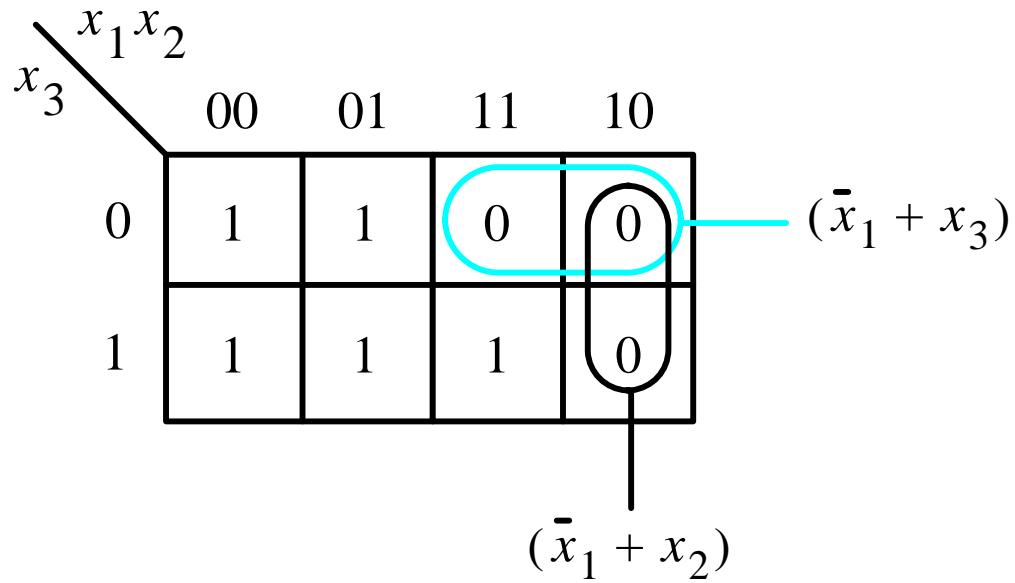


Figure 4.13. POS minimization of  $f(x_1, x_2, x_3) = \prod M(4, 5, 6)$ .

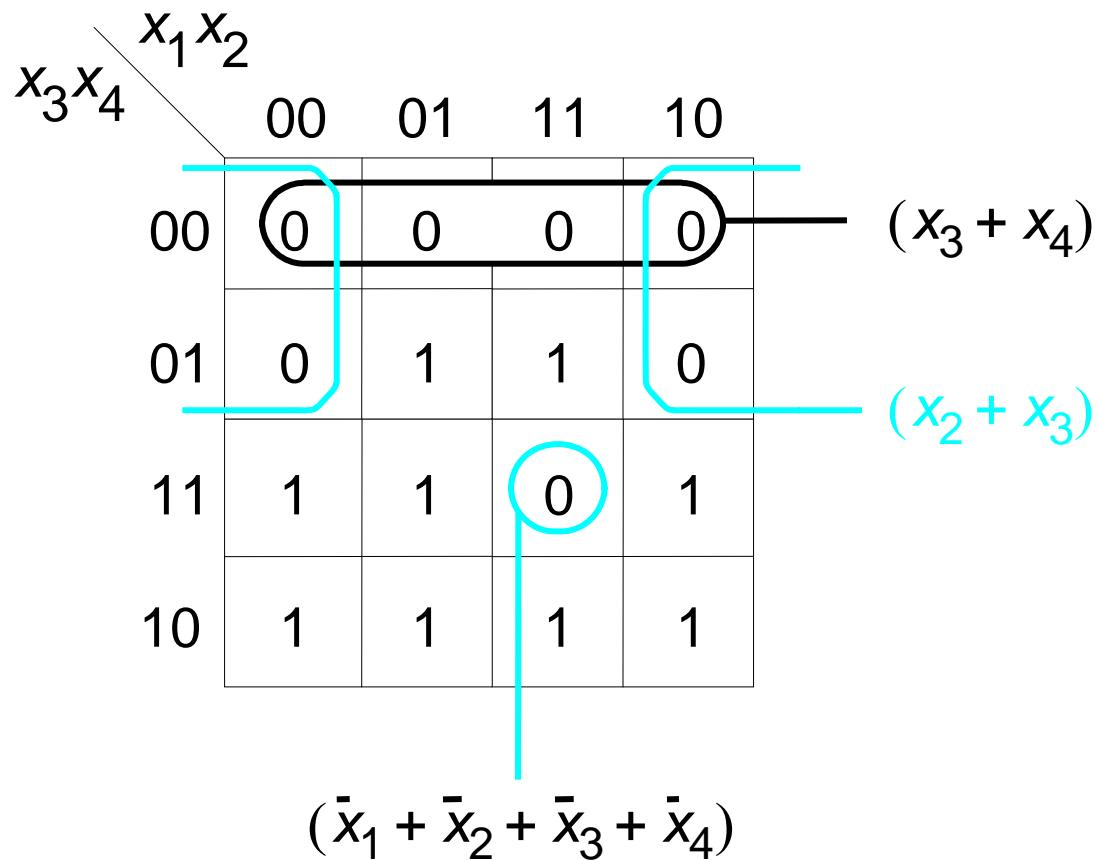


Figure 4.14. POS minimization of  $f( x_1, \dots, x_4 ) = \prod M(0, 1, 4, 8, 9, 12, 15)$ .

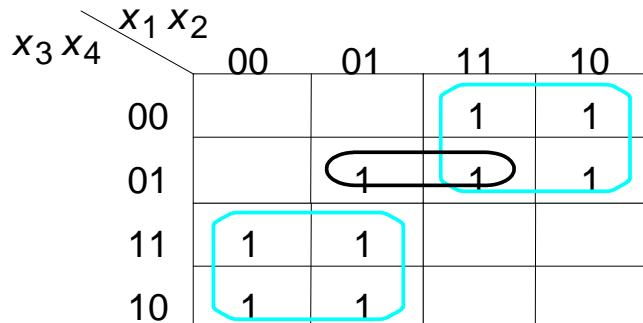
$x_3 \ x_4$	$x_1 \ x_2$	00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

(a) SOP implementation

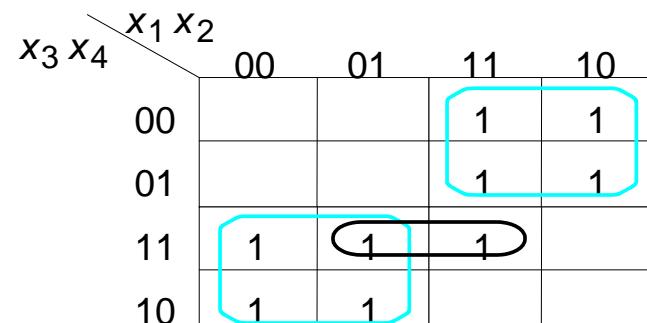
$x_3 \ x_4$	$x_1 \ x_2$	00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

(b) POS implementation

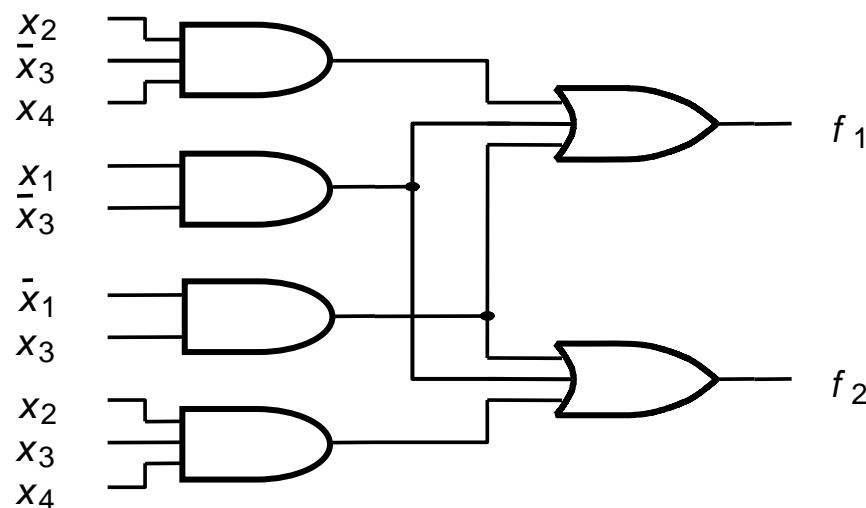
Figure 4.15. Two implementations of the function  $f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$ .



(a) Function  $f_1$

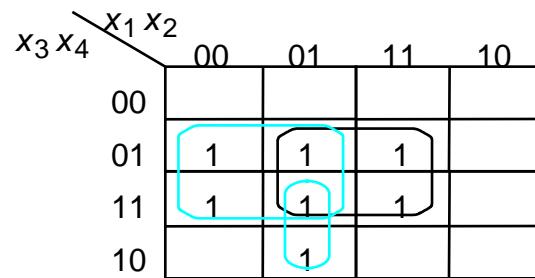


(b) Function  $f_2$

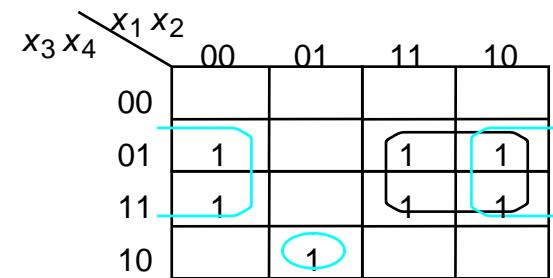


(c) Combined circuit for  $f_1$  and  $f_2$

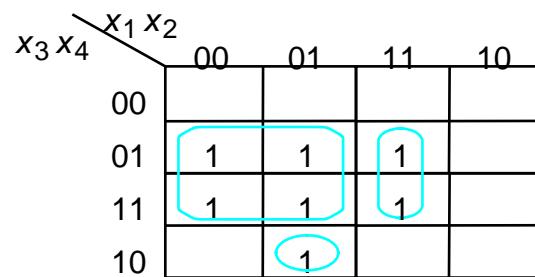
Figure 4.16. An example of multiple-output synthesis.



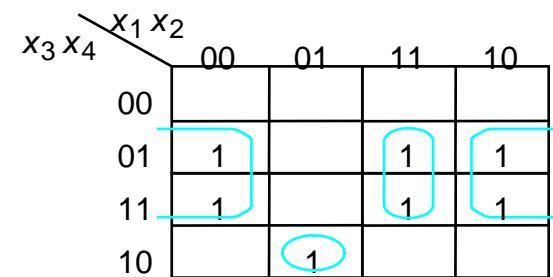
(a) Optimal realization of  $f_3$



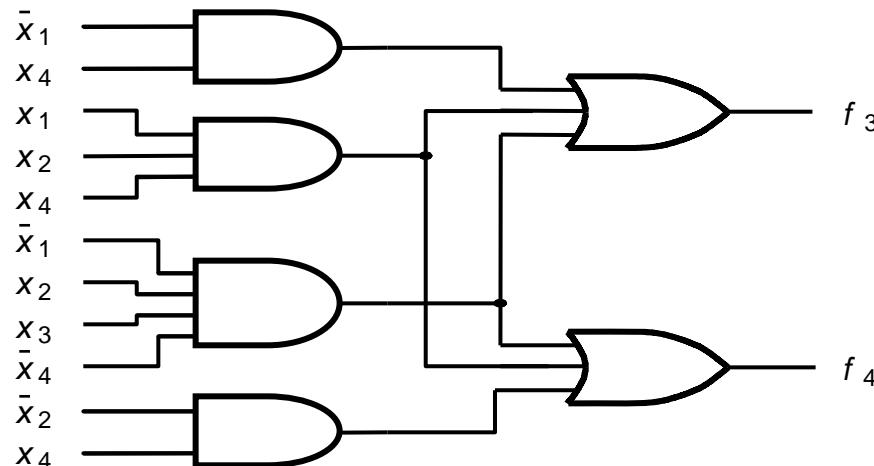
(b) Optimal realization of  $f_4$



(c) Optimal realization of



$f_3$  and  $f_4$  together



(d) Combined circuit for  $f_3$  and  $f_4$

Figure 4.17. An example of multiple-output synthesis.