Exemplo $y_1, \dots, y_n$ são n valores observados.

A distribuição de (X_1, X_2) tem densidade

$$f(x_1, x_2) \propto x_2^{n/2} \times \exp\left(-\frac{x_2}{2} \cdot \sum_{i=1}^{n} (y_i - x_1)^2\right) \exp\left(-\frac{x_1^2}{2}\right) x_2 e^{-x_2},$$

com $x_1 \in \mathbb{R}$ e $x_2 \in (0, \infty)$.

Daí obtemos

$$f_1(x_1) = f(x_1 | x_2) \propto \exp\left(-\frac{x_2}{2} \sum_{i=1}^{n} (y_i - x_1)^2\right) \exp\left(-\frac{x_1^2}{2}\right)$$

$$= \exp\left(-\frac{x_2}{2} \times \left\{\sum_{i=1}^{n} y_i^2 - 2x_1 \sum_{i=1}^{n} y_i + n x_1^2\right\}\right) \times \exp\left(-\frac{x_1^2}{2}\right)$$

$$= \exp\left(-\frac{x_2}{2} \cdot \sum_{i=1}^{n} y_i^2 + x_1 x_2 n\bar{y} - n\frac{x_2 x_1^2}{2} - \frac{x_1^2}{2}\right)$$

$$\propto \exp\left(x_1 x_2 n\bar{y} - n\frac{x_2 x_1^2}{2} - \frac{x_1^2}{2}\right)$$

$$= \exp\left(x_1 x_2 n\bar{y} - \frac{x_1^2}{2}(1 + n x_2)\right)$$

$$= \exp\left(-\frac{(1 + n x_2)}{2} \times \left\{x_1^2 - \frac{2x_1 x_2 n\bar{y}}{1 + n x_2}\right\}\right)$$

$$= \exp\left(-\frac{(1 + n x_2)}{2} \times \left\{x_1^2 - \frac{2x_1 x_2 n\bar{y}}{1 + n x_2} + \left(\frac{x_2 n\bar{y}}{1 + n x_2}\right)^2 - \left(\frac{x_2 n\bar{y}}{1 + n x_2}\right)^2\right\}\right)$$

$$\propto \exp\left(-\frac{1}{2(1 + n x_2)^{-1}} \times \left(x_1 - \frac{x_2 n\bar{y}}{1 + n x_2}\right)^2\right)$$

$$\Rightarrow X_1 | X_2 = x_2 \sim N\left(\frac{x_2 n\bar{y}}{1 + n x_2}, \frac{1}{1 + n x_2}\right)$$

Também obtemos

$$f_2(x_2) = f(x_2|x_1) \propto x_2^{n/2} \cdot \exp\left(-\frac{x_2}{2} \cdot \sum_{i=1}^{n} (y_i - x_1)^2\right) x_2 e^{-x_2}$$

$$= x_2^{\frac{n}{2}+1} \cdot \exp\left(-x_2 \left\{ 1 + \frac{1}{2} \sum_{i=1}^{n} (y_i - x_1)^2 \right\}\right).$$

$$\Rightarrow X_2 | X_1 = x_1 \sim \text{gama}\Big(\text{forma} = \frac{n}{2} + 2,$$

$$\text{taxa} = 1 + \frac{1}{2} \sum_{i=1}^{n} (y_i - x_1)^2\Big).$$

Dados: $\underset{\sim}{y} = (7,1;\ 6.9;\ 10.1;\ 8.9;\ 9.8;\ 10.3)$